UNIVERSITY OF TORONTO

DEPARTMENT OF ECONOMICS

ECONOMICS 381H1F – SUMMER 2013

**MANAGERIAL ECONOMICS II: PERSONNEL ECONOMICS**

# Midterm

# Version A

# SOLUTIONS

**Instructions**

The test is 50 minutes long. Non-programmable calculators are allowed. The test consists of four questions, each worth 5 points. Show all your work in the space provided below the question. If you need additional space, you may write on the back of the page.

**LAST NAME** ______________________________

**FIRST NAME** ______________________________

**STUDENT NUMBER** ______________________________

Good luck!

| Question 1 | Question 2 | Question 3 | Question 4 | Total |
|---|---|---|---|---|
| /5 | /5 | /5 | /5 | /20 |

1. Consider a relationship where the publisher contracts the writer to write a book. The publisher is risk neutral while the writer is risk averse with the coefficient of absolute risk aversion equal to 4. Suppose that the number of books sold depends stochastically on the writer's effort according to $q=\ln(e)+u$, where e is the writer's effort that can be observed and verified by the publisher and u is a random variable with a mean of zero and a variance of 1. The writer's cost of effort is given by $c(e)=e$. The outside option for both the publisher and the writer is 0. Assume that the writer's pay is linear in the number of books sold (i.e. $w=a+bq$).

    a. (1 point) Write down the expected payoff (i.e. certainty equivalent) for the publisher and the writer using the information given in the question.
    b. (2 points) What is the risk premium for each of the publisher and the writer if they optimally share risk in this relationship?
    c. (2 points) Is it efficient to form this relationship?

a. The publisher's expected payoff is given by E[q]-E[w] given that the publisher is risk neutral.. Given that $q=\ln(e)+u$ and that $E[u]=0$, $E[q]=\ln(e)$. Also, given that $w=a+bq$, $E[w]=a+b\ln(e)$. Therfore, $E[V]=\ln(e)-a-b\ln(e) = (1-b)\ln(e)-a$. For the writer, the expected payoff is E[w]-c(e)-RP given that the writer is risk-averse. Again, $E[w]=a+b\ln(e)$ and $c(e)=e$ from the question. The risk premium is $0.5rVar[w]=0.5(4)Var[a+b\ln(e)+bu]=2b^2Var[u]=2b^2$ since $Var[u]=1$ from the question. Therefore, $E[U]=a+b\ln(e)-e-2b^2$.

b. The optimal risk sharing requires that the party that is more risk-averse should bear more risk. Since the publisher is risk neutral while the writer is risk neutral, this implies that the publisher should fully insure the writer. This could be accomplished by paying the writer a fixed salary, that is, with b=0. In this case, the writer's risk premium is $2b^2=0$, while the publisher's risk premium is always zero since he is risk neutral.

c. It is efficient to form the relationship if $E[U]+E[V]-R-S\geq0$. Given than R=S=0 from the problem, this implies that $E[U]+E[V]\geq0$ for the relationship to form. Now, the optimal effort level, given $E[q]=\ln(e)$ and $c(e)=e$ gives us $1/e^*-1=0$, or $e^*=1$. Also, we have that $E[V]+E[U]= (1-b)\ln(e)-a +a+b\ln(e)-e-2b^2=\ln(e)-e-2b^2$. Given $e^*=1$ and $b^*=0$, this yields $\ln(1)-1-2(0)^2=-1<0$. Therfore, it is not efficient to form the relationship.

2. You wish to hire an accountant to help you find legal savings in your tax return. For each unit of effort e, the accountant can increase your savings q by e+u, where u is a random variable with a mean of 0 and a variance of 1. The accountant's cost of effort is $0.5e^2$, her outside option is 0, and her coefficient of absolute risk aversion is 2. You are risk neutral and your outside option is 0. Suppose that you offer the accountant a retainer (a) plus an additional payment (bq) that depends on the actual savings found.

   a. (1 point) Write down the expected payoff (i.e. certainty equivalent) for you and the accountant using the information given in the question.
   b. (2 points) What is your maximum expected payoff if you can observe the accountant's effort?
   c. (2 points) What is your maximum expected payoff if you cannot observe the accountant's effort?

a. Since you are the principal and you are also risk neutral, your expected payoff is E[q]-E[w]. Given q=e+u and E[u]=0, this gives us E[q]=e. Further, w=a+bq, so E[w]=a+be. Therefore, E[V]=E[q-w]=(1-b)e-a. For the accountant, the expected payoff is E[U]=E[w]-c(e)-RP since the accountant is risk averse. We have, as before, E[w]=a+be, c(e)= $0.5e^2$ from the question, and the risk premium is 0.5rVar[w]=0.5(2)Var[a+be+bu]=(1)Var[bu]=$b^2$. Therefore, E[U]=a+be-$0.5e^2$ –$b^2$.

b. If you can observe the accountant's effort, you can provide an optimal incentive and insurance contract. The optimal insurance requires you to fully insure the accountant since you are risk neutral and the accountant is risk averse, so b*=0. Further, the optimal effort level maximizes E[q]-c(e) = e-$0.5e^2$, which yields e*=1 from the first-order condition. Lastly, the accountant's participation constraint, given b*=0 and e*=1, is equal to E[U]=a+be-$0.5e^2$ –$b^2$=a-0.5≥0, so a*=0.5. Therefore, your expected payoff is E[q-w]=e*-a*=1-0.5=0.5.

c. If you cannot observe the accountant's effort, you can specify the contract based on only a and b, but not e. The account will choose e to maximize his expected payoff E[U]. The first-order condition for e is then b=e. This is the incentive compatibility constraint. The participation constraint is then E[U]=a+be-$0.5e^2$ –$b^2$= a+ $b^2$-$0.5b^2$ –$b^2$=a-$0.5b^2$=0 so a=$0.5b^2$. Your expected payoff is then (1-b)e-a=(1-b)b-$0.5b^2$=b-$b^2$-$0.5b^2$=b-$1.5b^2$. The first-order condition for b is then 1-3b=0, or b*=1/3. Therefore, your expected payoff is b-$1.5b^2$=(1/3)-(1.5)$(1/3)^2$=1/6.

3. In their research, Gibbons and Murphy (1990) used data on about 2,000 Chief Executive Officers from about 1,300 firms between 1974 and 1986. Their main result can be expressed using the following regression model: $E[w] = 0.068 + 0.1805q - 0.1490y$, where w represents the CEO's pay (in logarithm), q is the firm's rate of return, and y is the market rate of return. The t-statistics were 21.0 and -7.6 for the coefficients on q and y, respectively.

   a. (2 points) What is the main hypothesis that Gibbons and Murphy test in their study? How does this hypothesis relate to the theory of using multiple signals in principal-agent relationships?
   b. (1 point) What is the assumption required to allow us to interpret this regression model as a causal impact of q and y on w?
   c. (2 points) Interpret the coefficient estimates on q and y in terms of their sign and statistical significance.

a. Gibbons and Murphy test the hypothesis that using additional performance signals (the market rate of return in their study) may impact the agent's productivity, as reflected in their pay. The principal-agent theory suggests that the principal should use any signal informative about the agent's performance (the informativeness principle) and that the signal should be used in the opposite way of the correlation between this signal and the agent's actual performance. In this study, the market return and the firm's rate of return are expected to be positively correlated; therefore, we expect that the sign on the market return coefficient will be negative.

b. The assumption is that the only relevant differences between CEOs in different firms are the differences in the firm's rate of return and the market's rate of return. If this is the case, then these differences across firms can be interpreted as having a causal impact on the CEO's performance.

c. The coefficient on q, the firm's own rate of return, is positive and statistically significant (t-statistic > 2). This is as expected, as we would expect that the pay should be positively related to the firm's own performance. The coefficient on y, the additional signal of the market rate of return, is negative and statistically significant (t-statistic < -2). This is consistent with the principal-agent theory given that we expect the firm's and market rate of return to be positively correlated and that the optimal use of additional signals is opposite of the correlation between the signal and the agents' outcome.

4. Chef Boyardee wishes to hire a manager for his new restaurant. Chef Boyardee proposes to pay w=a+bq+cy, where q is the number of customers in his restaurant and y is the number of customers in a restaurant of similar quality. Specifically, Chef Boyardee knows that E[q]=e, Var[q]=2, E[y]=0, Var(y)=1, and cov(q,y)=0.9, where e is the manager's effort that Chef Boyardee cannot observe. However, Chef Boyardee knows that the manager's cost of effort is $0.5e^2$ and his coefficient of risk aversion is r=2. On the other hand, Chef Boyardee is risk neutral. Assume that both parties have outside option of zero.

   a. (1 point) Write down the expected payoff (i.e. certainty equivalent) for Chef Boyardee and the manager using the information given in the question.
   b. (1 point) Write down the manager's participation constraint.
   c. (1 point) Write down the manager's incentive compatibility constraint.
   d. (2 points) Find b and c that maximize the Chef Boyardee's expected payoff.

a. The expected payoff for Chef Boyardee is E[q]-E[w] since the chef is risk-neutral. Now, this gives E[V]=e-a-be since E[q]=e and E[y]=0. Further, the manager's expected payoff is E[U]=E[w]-c(e)-RP since the manager is risk averse. Now, E[w]=a+be since E[q]=e and E[y]=0, $c(e)=0.5e^2$ from the question, and $RP=0.5rVar[w]=0.5(2)Var[a+bq+cy]=b^2(2)+c^2(1)+2bc(0.9)=2b^2+c^2+1.8bc$. Therefore, $E[U]=a+be-0.5e^2-2b^2-c^2-1.8bc$.
b. The manager's participation constraint is E[U]=0, or $a+be-0.5e^2-2b^2-c^2-1.8bc=0$.
c. The manager maximizes his expected payoff by choosing the effort level. The first-order condition is b-e=0.
d. Given the participation constraint and the incentive compatibility constraint, the expected payoff for Chef Boyardee is $E[V]=e-a-be = b - 0.5b^2-2b^2-c^2-1.8bc$. The first-order conditions for b and c are, respectively, 1-b-4b-1.8c=0 and -2c-1.8b=0. Solving the second equation for c yields c=-0.9b. Substituting for c in the first equation then yields 1-b-4b-1.8(-9b)=0. Solving this equation for b yields $b^*\approx0.3$. Substituting for b in c=-0.9b finally gives us $c^*\approx-0.27$.